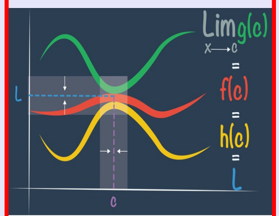
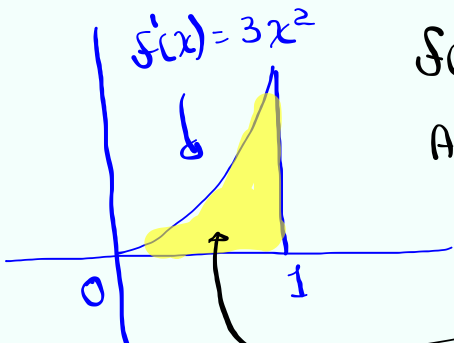
Calculus I

Lecture 43



Feb 19-8:47 AM



$f(x) = x^3 + C$

Area $= f(1) - f(0)$

$= (1^3 + C) - (0^3 + C)$

$= \boxed{1}$

Verify this by using

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \cdot \Delta x$$

$f(x) = 3x^2$

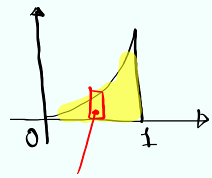
$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

$x_i = a + i\,\Delta x$

$= i \cdot \frac{1}{n}$

Nov 14-8:33 AM

$f(x)=3x^2$



$a=0\ ,\ b=1$

$\Delta x=\frac{b-a}{n}=\frac{1}{n}$

$x_i=a+i\Delta x=0+i\cdot\frac{1}{n}=\frac{i}{n}$

$f(x_i)=3x_i^2=3\left(\frac{i}{n}\right)^2=\frac{3i^2}{n^2}$

$A_i=f(x_i)\cdot\Delta x=\frac{3i^2}{n^2}\cdot\frac{1}{n}=\frac{3}{n^3}\cdot i^2$

$A=\lim_{n\to\infty}\sum_{i=1}^{n}A_i$

$=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n^3}\cdot i^2$

$=\lim_{n\to\infty}\frac{3}{n^3}\cdot\sum_{i=1}^{n}i^2$

$=\lim_{n\to\infty}\frac{3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

$=\lim_{n\to\infty}\frac{6n^3+?n^2+?n}{6n^3}=\frac{6}{6}=\boxed{1}$

Divide by n^3

Nov 18-7:26 AM

Find abs. Max & abs. Min of $f(x)=2x-x^2$ on $[0,3]$.

Int. Algebra / Algebra II } College Algebra / PreCalc } Parabola opens down, Vertex $(1,1)$

Evaluate at endpoints.

$f(0)=0$, $f(3)=-3$ ← Min.

Find C.P. on $[0,3]$

$f(0)=0\ ,\ f(3)=-3$

$f'(x)=2-2x$

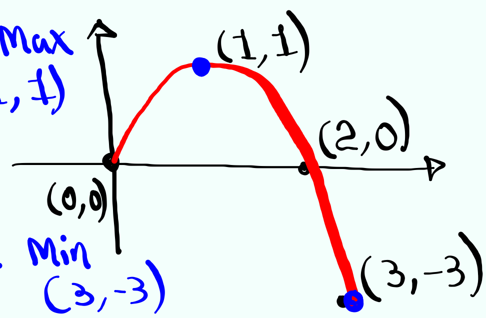
$f'(x)=0\qquad 2-2x=0$

$x=1$

$\boxed{f(1)=1}$ ← Max.

Abs. Max $(1,1)$

Abs. Min $(3,-3)$



Nov 18-7:33 AM

Find Abs. Max & Abs. Min. of

$f(x) = 5 + 54x - 2x^3$ on $[0,4]$.

$\boxed{f(0) = 5}$

$\boxed{f(4) = 93}$

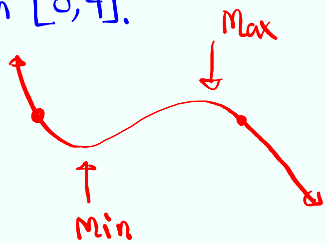
$f'(x) = 54 - 6x^2$

$f'(x) = 0$

$54 - 6x^2 = 0$

$x^2 = 9$

$x = \pm 3$ $\quad x = 3 \rightarrow \boxed{f(3) = 113}$



Abs. Min at $(0,5)$

Abs. Max at $(3,113)$.

Find Abs. Max and Abs. Min of

$f(x) = \dfrac{x}{x^2 - x + 1}$ on $[0,3]$.

Nov 18-7:40 AM

Introduction to Integration:

$$\int f'(x)\,dx = f(x) + C$$

integral

Respect to x

Integrand

$$\frac{d}{dx}\left[\text{Answer}\right] = \text{Integrand}$$

$\int 3x^2\,dx = x^3 + C$ $\qquad \dfrac{d}{dx}\left[x^3 + C\right] = 3x^2$

$\int \cos x\,dx = \sin x + C$ $\qquad \dfrac{d}{dx}\left[\sin x + C\right] = \cos x$

$\int \sin x\,dx = -\cos x + C$ $\qquad \dfrac{d}{dx}\left[-\cos x + C\right] = -(-\sin x) + 0$

$= \sin x$

Nov 18-7:47 AM

Power Rule

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad , n \neq -1$$

$$\int \frac{1}{x^4} \, dx = \int x^{-4} \, dx = \frac{x^{-4+1}}{-4+1} + C$$

$$= \frac{x^{-3}}{-3} + C = \boxed{\frac{-1}{3x^3} + C}$$

Nov 18-7:53 AM

$$\int \sqrt{x} \, dx = \int x^{1/2} \, dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{x^{\frac{3}{2}}}{3/2} + C = \boxed{\frac{2}{3} x\sqrt{x} + C}$$

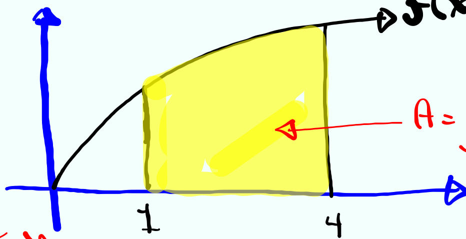
If $f(x) \geq 0$ on $[a, b]$, then the area below $f(x)$, above x-axis is given by

$$\int_a^b f(x) \, dx = F(x) \Big|_a^b = F(b) - F(a)$$

where $\frac{d}{dx}\left[F(x)\right] = f(x)$

Nov 18-7:56 AM

Find the area below $f(x) = \sqrt{x}$, above x-axis on $[1, 4]$.



$f(x) = \sqrt{x}$

$A = \int_1^4 \sqrt{x}\, dx$

see last example

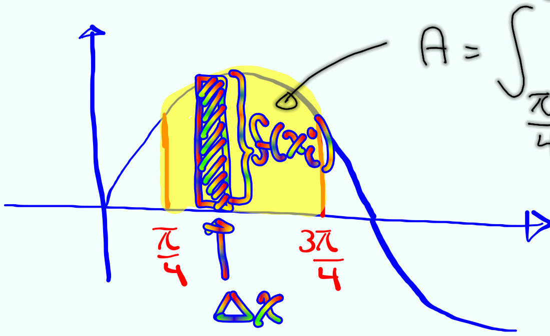
$= \left[\frac{2}{3} x\sqrt{x} + C\right]\Big|_1^4 = \left[\frac{2}{3}\cdot 4\sqrt{4} + \cancel{C}\right] - \left[\frac{2}{3}\cdot 1\sqrt{1} + \cancel{C}\right]$

$= \frac{2}{3}\cdot 8 - \frac{2}{3}\cdot 1$

$= \frac{2}{3}(8-1) = \frac{2}{3}\cdot 7 = \boxed{\frac{14}{3}}$

Nov 18-8:02 AM

Find the area below $f(x) = \sin x$, above x-axis on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$



$A = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin x\, dx$

$= (-\cos x + C)\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

$= \left[-\cos\frac{3\pi}{4} + \cancel{C}\right] - \left[-\cos\frac{\pi}{4} + \cancel{C}\right] = -\cdot\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$

Nov 18-8:07 AM

Find all points on $4x^2+y^2=4$ that are farthest from $(1,0)$.

$\rightarrow y^2=4-4x^2$

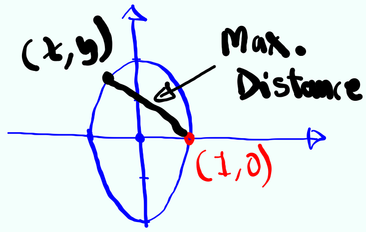
$\frac{x^2}{1}+\frac{y^2}{4}=1$

Int. Algebra
Algebra II
College Algebra
Precalc.

Ellipse
$(0,0)$
± 1 horizontally
± 2 vertically

$d=\sqrt{(x-1)^2+(y-0)^2}$

$d=\sqrt{\underbrace{(x-1)^2+y^2}_{\text{Max.}}}$



$f(x)=(x-1)^2+4-4x^2$

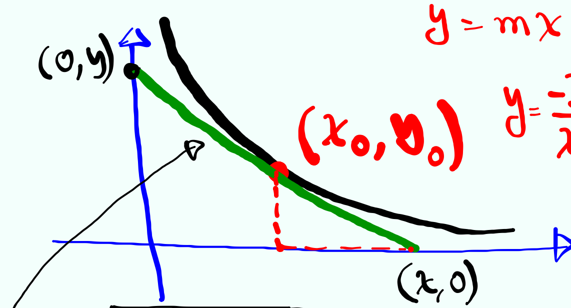
$f'(x)$, $f'(x)=0$, $f''(x)$, $f''(\text{C.P.})$

Nov 18-8:13 AM

what is the shortest possible length of a line segment in the first quadrant that is tangent to the curve $y=\frac{3}{x}$ at some point?

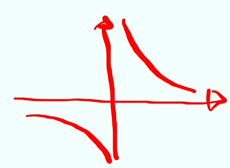
$y=mx+b$

$y=\frac{-3}{x_0}x+b$



$L=\sqrt{(0-x)^2+(y-0)^2}=\sqrt{x^2+y^2}$

Recall $y=\frac{1}{x}$



$y'=\frac{-3}{x^2}$

Nov 18-8:22 AM